Mathematisches Institut
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Universität zu Köln
January 21st 2015

Algebraic Topology – Homework 13

Due date : January 28th in class

Exercise 1.

Exercise number 9 on page 205 of Hatcher's book.

Exercise 2.

Let X be any topological space whose homology groups are finitely generated. For each n consider a decomposition of $H_n(X)$ into a direct sum of cyclic groups, and define τ_n as the number of finite cyclic groups of even order in this decomposition.

(a) Give an explicit expression (with proof) of the cohomology groups of X with $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ coefficients in terms of the Betti numbers of X.

(b) Give an explicit expression (with proof) of the cohomology groups of X with $\mathbb{Z}_2$ coefficients in terms of the Betti numbers of X and the τ_n's.

Exercise 3.

Let G be an abelian group. Compute the cohomology groups with coefficients in G of $\mathbb{C}P^2$ and $S^2 \vee S^4$. Are these two spaces homeomorphic ?

Exercise 4.

Exercise number 1 on page 267 of Hatcher's book (Hint : you will need to use somehow the characteristic of the field F).

Exercise 5.

Let $f\colon S^2 \longrightarrow T^2$ be a continuous map from the 2-sphere to the 2-torus T. Show that the induced map in cohomology $f^*\colon H^2(T^2) \longrightarrow H^2(S^2)$ is trivial, and conclude that the map in homology $f_*\colon H_2(S^2) \longrightarrow H_2(T^2)$ is trivial as well. Can you find a continuous map $g\colon T^2 \longrightarrow S^2$ such that the induced map in homology $g_*\colon H_2(T^2) \longrightarrow H_2(S^2)$ is non-trivial ?